

CPM Study Notes

Robert Perrine, PMP

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IMAGES FOR THE CPM PRESENTATION

CPM QUICK REFERENCE

Pick ONE Style and stay with it

For Me, the easiest convention is:

Starting Point: Zero

Addition: Early Finish = Early Start + Duration

Increment: Early Start = Early Finish of LATEST predecessor

Forward Pass

Read the problem carefully

Draw the diagram with your notation

Add the values given by the Question

Work left to right on tasks with 1 predecessor

When tasks Join, Latest EF wins

Backward Pass

Convention going back matches forward

Start with the End Node

Work right to left

Late Finish = Late Start of EARLIEST dependent

Late Start = Late Finish - Duration

Critical Path

Project must have a single start and single end

Calculate the Floats

Float = Late Finish - Early Finish

Critical Path is the set of Nodes with 0 Float

Write down your answer

Answer the Question

Read the problem carefully -- again

Compare your answer to the list of answers

Then answer the question

WBSID	
Duration	Float
Early Start	Early Finish
Late Start	Late Finish

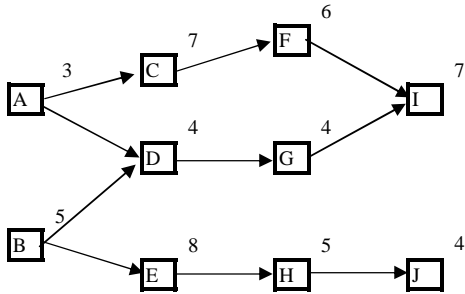
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<http://members.cox.net/rperrine5>

CPM PROBLEM 1 -- WORKED TWICE

Calculate the critical path for the network shown in the following diagram.



Syntax
 D is the duration
 N is the node identifier

Solution to Problem 1-A

Step 1

Choose one notational system and stay with it.

Here's the notational system that I prefer. I'll show another in the solution to Problem 1-B

WBSID	
Duration	Float
Early Start	Early Finish
Late Start	Late Finish

Step 2

Decide upon an expression format and stay with it.

I prefer to start with day=0, but you could also start with day=1.

I find it simpler to let the finish date for one task be the start date for the dependent tasks.

But, if you think about project scheduling tools, like MS Project, the start date for the dependent is the "next" day.

I'll use the simpler syntax in the solution to Problem 1-A and then use the alternate syntax in the solution to Problem 1-B.

Step 3

Set a starting task. In this example, there are two "starting points".

I prefer to then add a pseudo-node so that there is a single starting point.

The starting "day" for this task is day 0, and the duration for a pseudo-node is also 0.

Start	
0	
0	

Step 4

The end date for node "Start" is the "early start" + "duration".

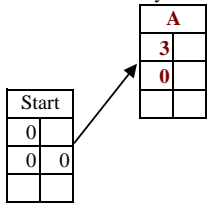
Start	
0	
0	0

CPM PROBLEM 1 -- WORKED TWICE

Step 5

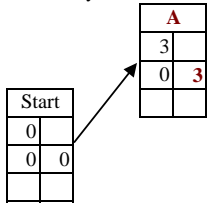
Add the "next" task, and record the duration.

Then set the "early start" = the "early finish" of it's "latest" predecessor.



Step 6

The "early finish" of this task is it's "early start" + "duration".

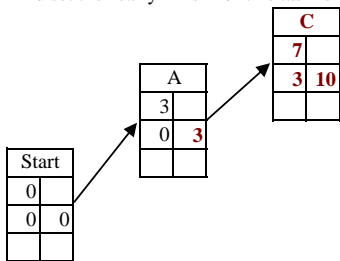


Step 7

Work towards the right, but only add nodes where ALL of the predecessors have already been defined.

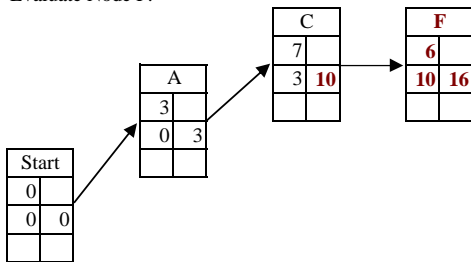
Remember: set the "early start" = the "early finish" of it's "latest" predecessor.

And set the "early finish" of this task is it's "early start" + "duration".



Step 8

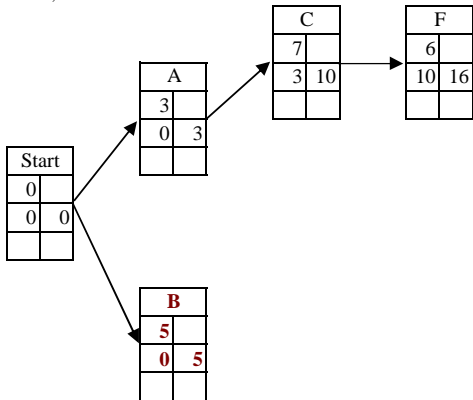
Evaluate Node F.



CPM PROBLEM 1 -- WORKED TWICE

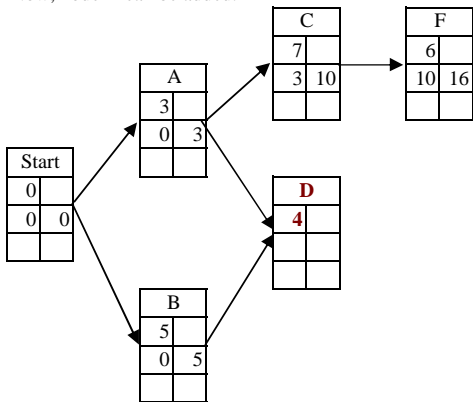
Step 9

Node I has a predecessor (G) that has not yet been defined, so it cannot yet be added.
 Node D has a predecessor (B) that has not yet been defined, so it cannot yet be added.
 Thus, node B needs to be evaluated next.



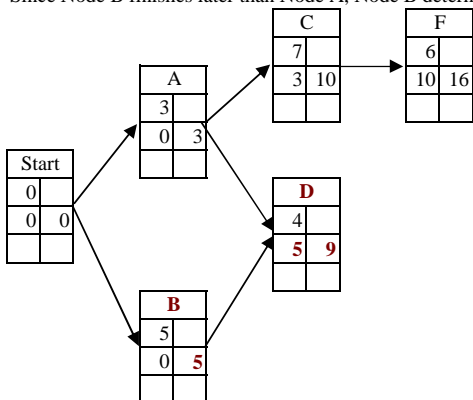
Step 10

Now, node D can be added.



Step 11

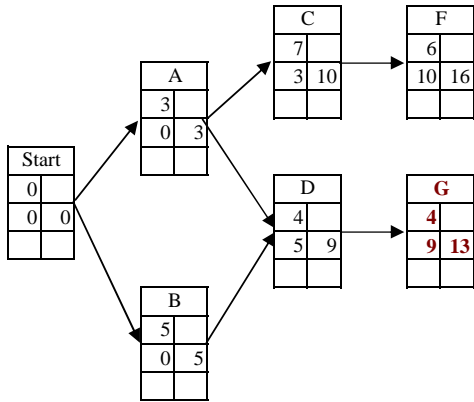
Node D has two predecessors.
 Since the relationships show are Finish-to-Start, Node D cannot start until both predecessors have finished.
 Since Node B finishes later than Node A, Node B determines the "early start" for Node D.



CPM PROBLEM 1 -- WORKED TWICE

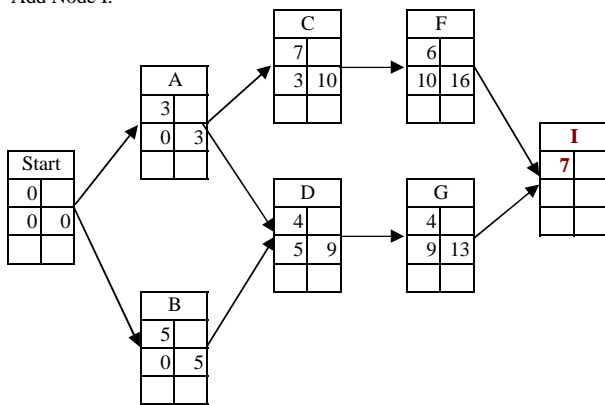
Step 12

Evaluate Node G.



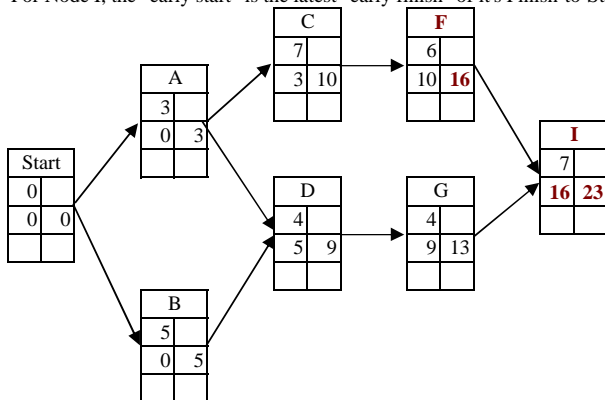
Step 13

Add Node I.



Step 14

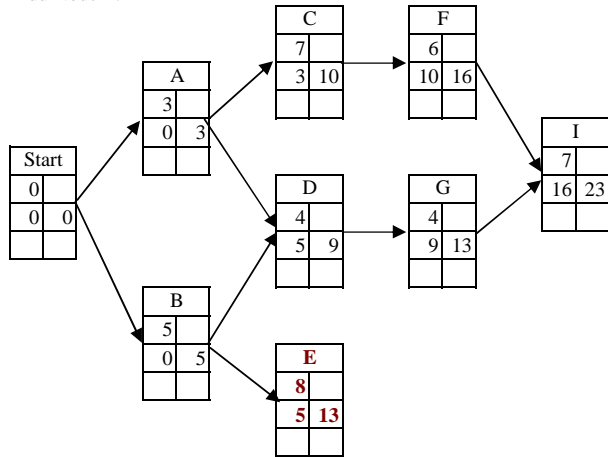
For Node I, the "early start" is the latest "early finish" of it's Finish-to-Start predecessors.



CPM PROBLEM 1 -- WORKED TWICE

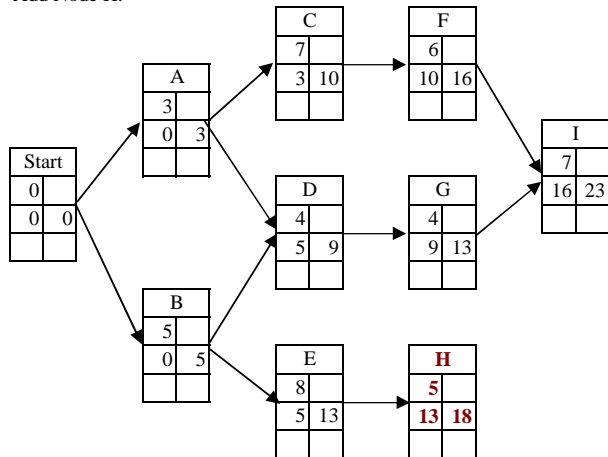
Step 15

Add Node E.



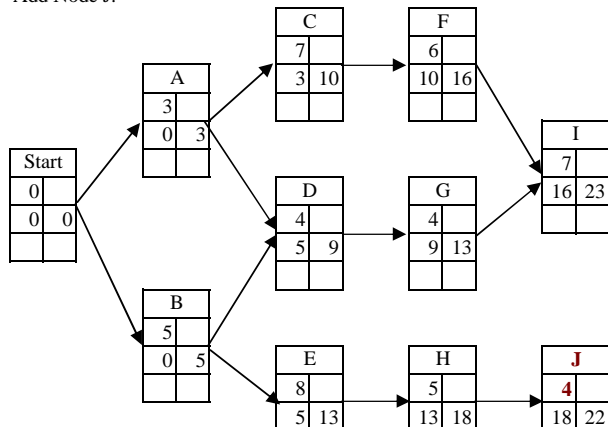
Step 16

Add Node H.



Step 17

Add Node J.



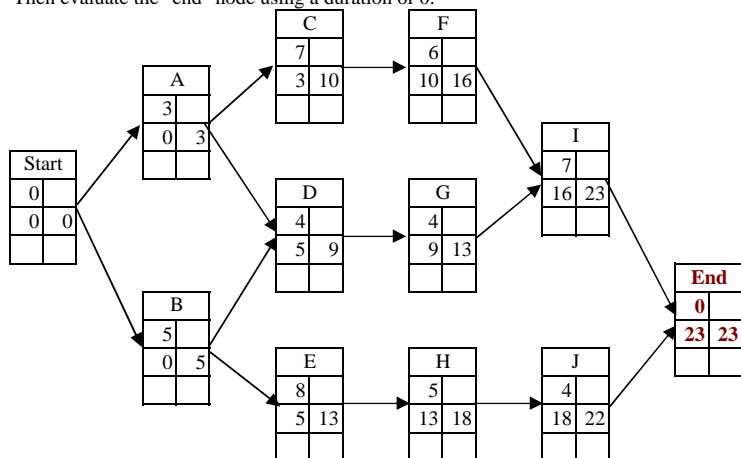
CPM PROBLEM 1 -- WORKED TWICE

Step 18

The "backward pass", like the "forward pass" needs a single start point.

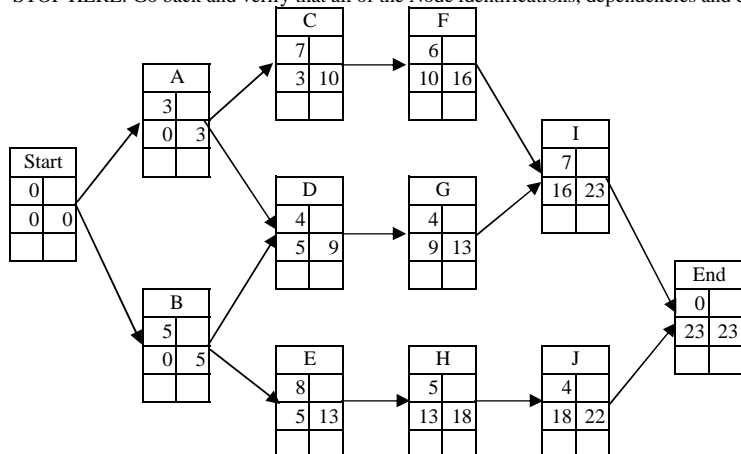
So add a pseudo-node for the common "end point".

Then evaluate the "end" node using a duration of 0.

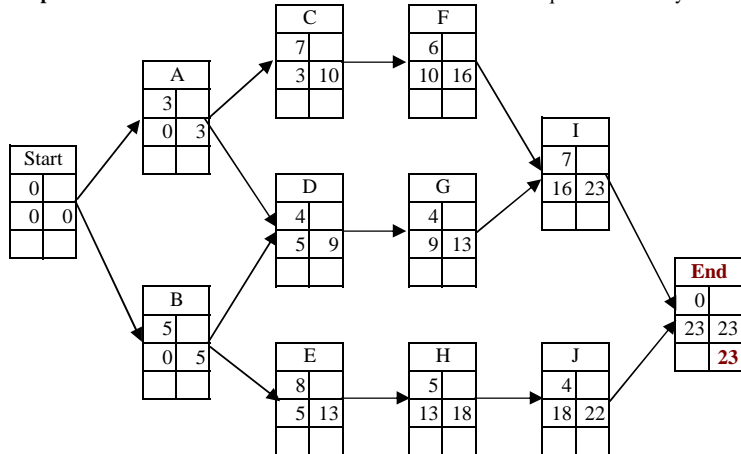


Step 19

STOP HERE: Go back and verify that all of the Node identifications, dependencies and durations have been copied correctly.



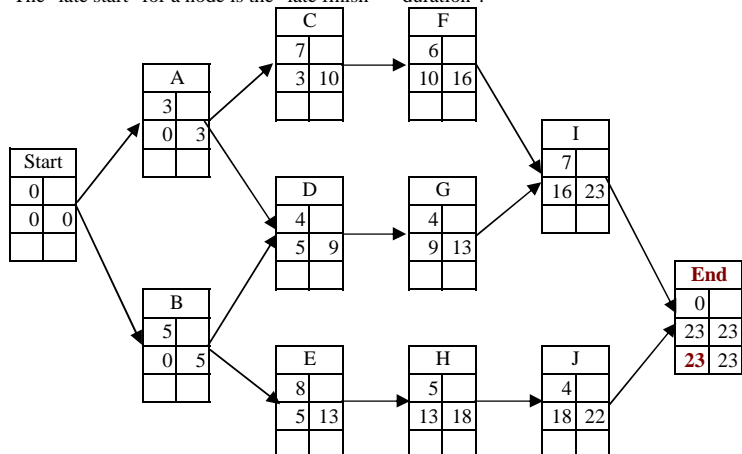
Step 20 The "late finish" for the final task in the network is equal to the "early finish".



CPM PROBLEM 1 -- WORKED TWICE

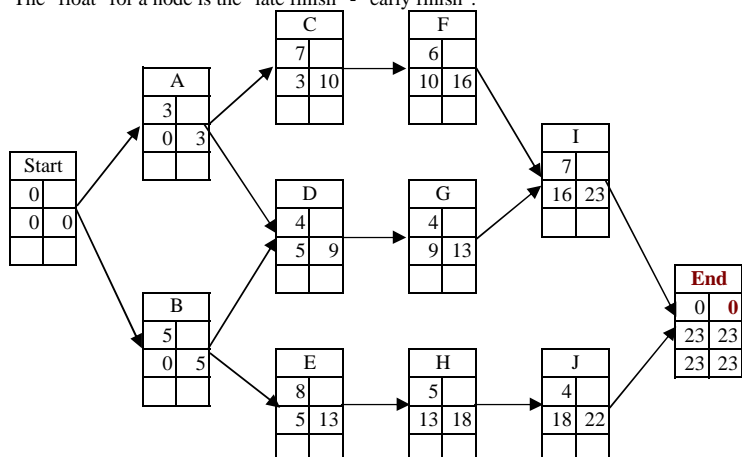
Step 21

The "late start" for a node is the "late finish" - "duration".



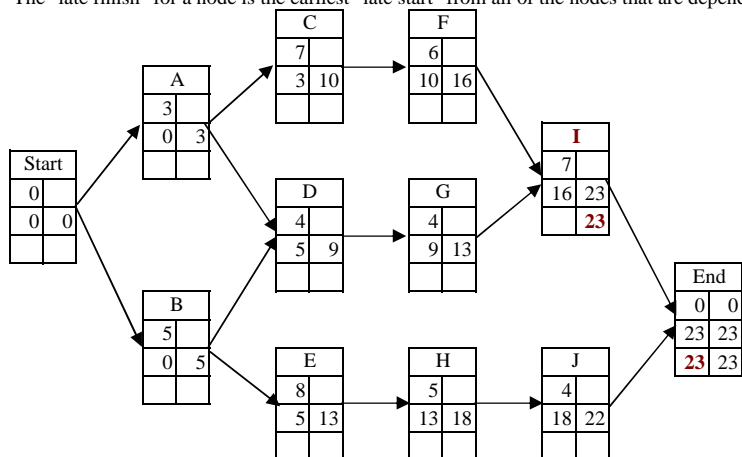
Step 22

The "float" for a node is the "late finish" - "early finish".



Step 23

The "late finish" for a node is the earliest "late start" from all of the nodes that are dependent on this node.

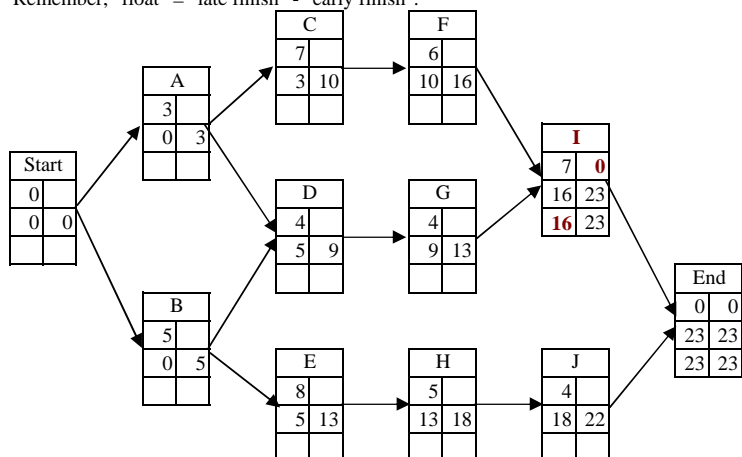


CPM PROBLEM 1 -- WORKED TWICE

Step 24

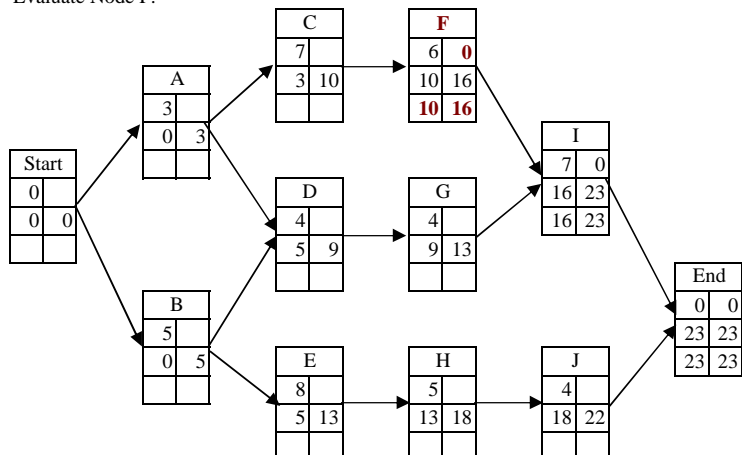
Remember, "late start" = "late finish" - "duration".

Remember, "float" = "late finish" - "early finish".



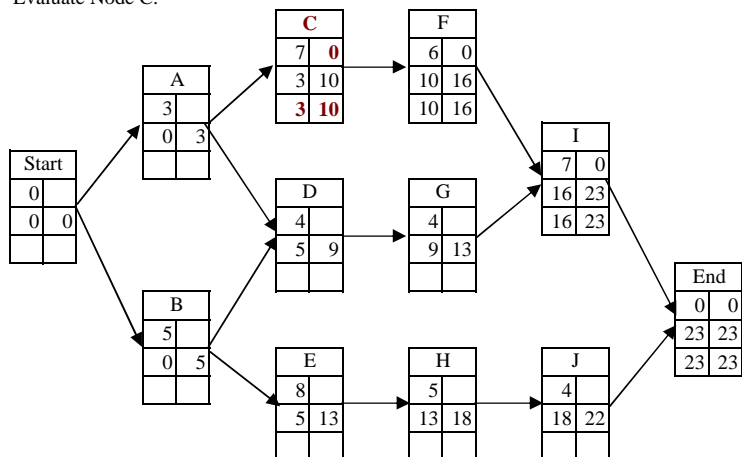
Step 25

Evaluate Node F.



Step 26

Evaluate Node C.



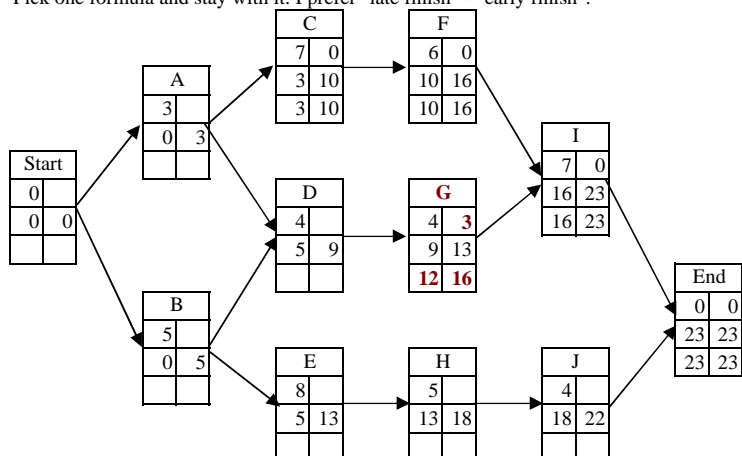
CPM PROBLEM 1 -- WORKED TWICE

Step 27

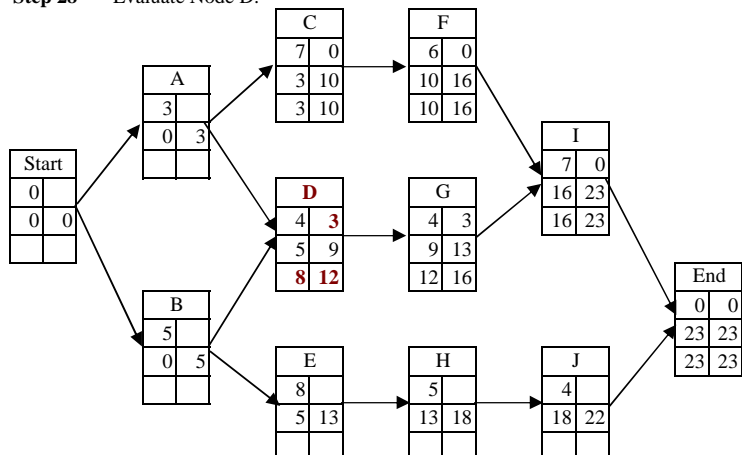
Node A cannot be evaluated until both Node C and Node D have been evaluated. So go back and evaluate Node G.

Note that an alternative formula for "float" is "late start" - "early start".

Pick one formula and stay with it. I prefer "late finish" - "early finish".

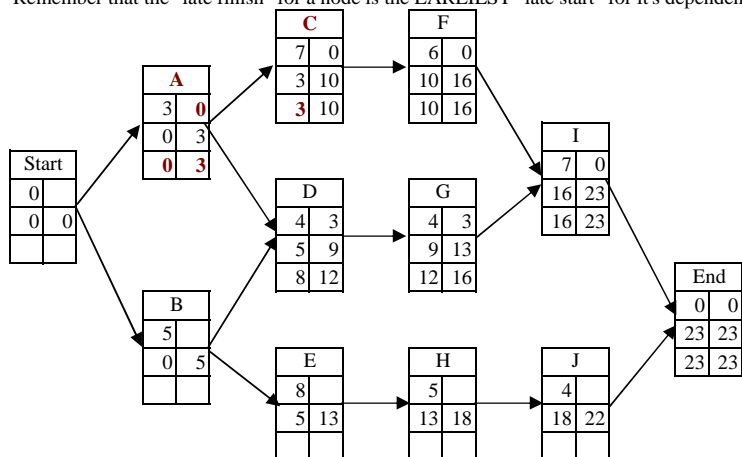


Step 28 Evaluate Node D.



Step 29 Evaluate Node A.

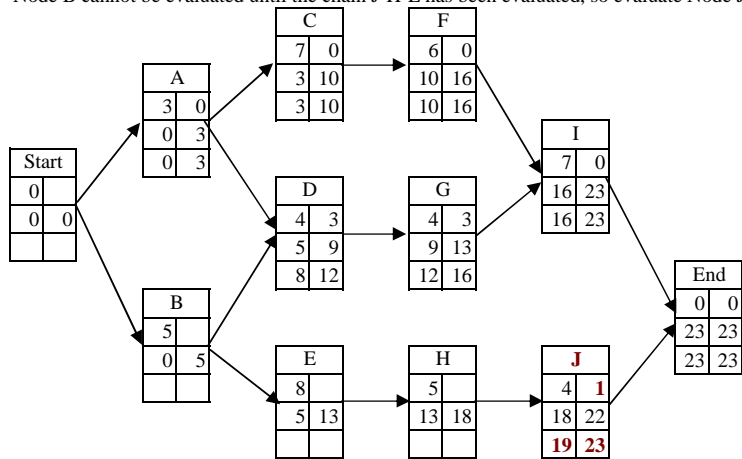
Remember that the "late finish" for a node is the EARLIEST "late start" for its dependents.



CPM PROBLEM 1 -- WORKED TWICE

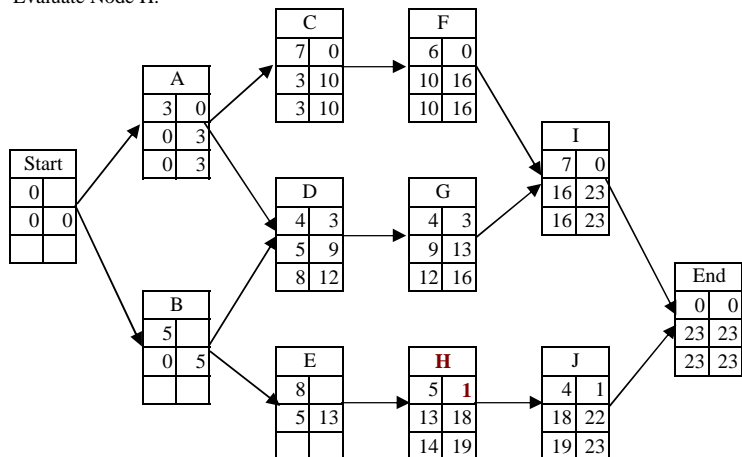
Step 30

Node B cannot be evaluated until the chain J-H-E has been evaluated, so evaluate Node J next.



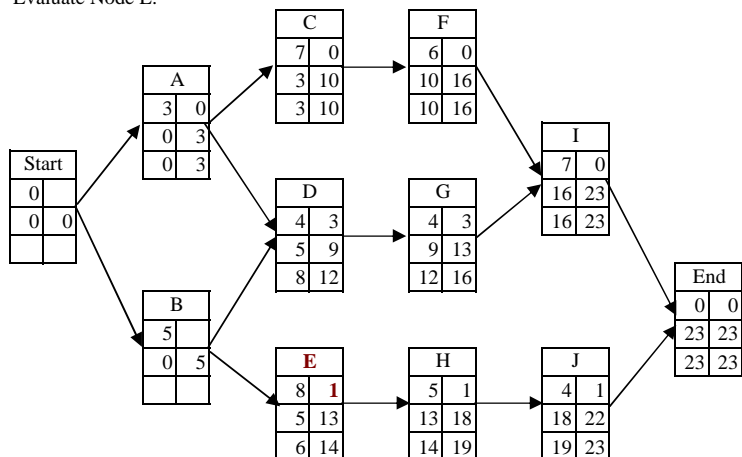
Step 31

Evaluate Node H.



Step 32

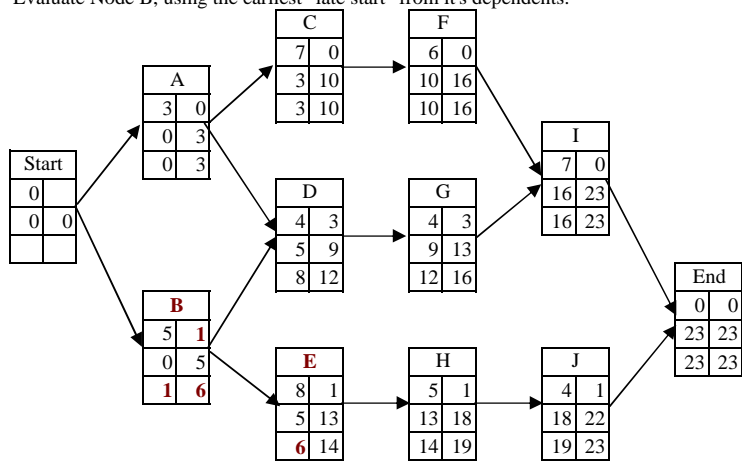
Evaluate Node E.



CPM PROBLEM 1 -- WORKED TWICE

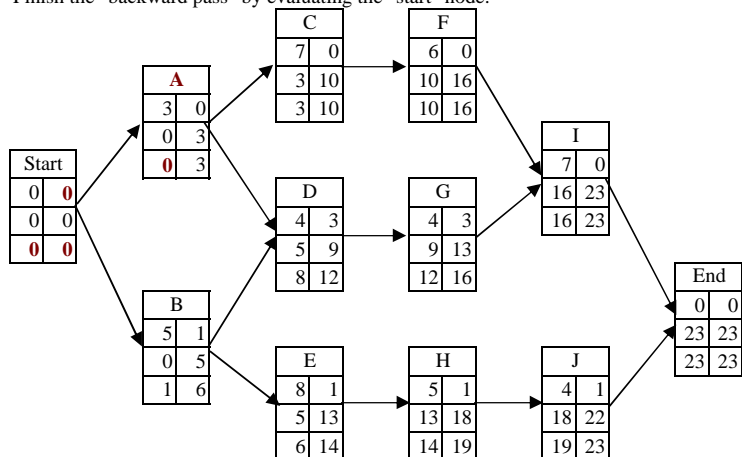
Step 33

Evaluate Node B, using the earliest "late start" from it's dependents.



Step 34

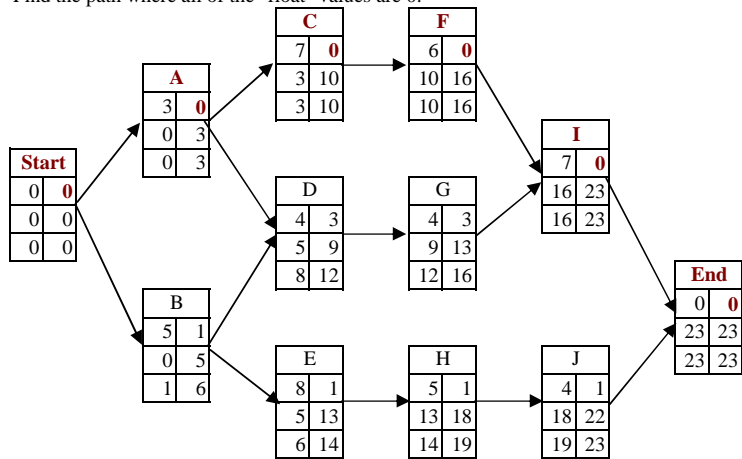
Finish the "backward pass" by evaluating the "start" node.



CPM PROBLEM 1 -- WORKED TWICE

Step 35

Find the path where all of the "float" values are 0.



Step 36

Write down the answer, ignoring the pseudo-nodes "start" and "end".

A → C → F → I

Step 37

Check the possible answers and find a match.

Double check the match. Does it have exactly the same number of nodes, the same beginning, the same ending and all the same values.

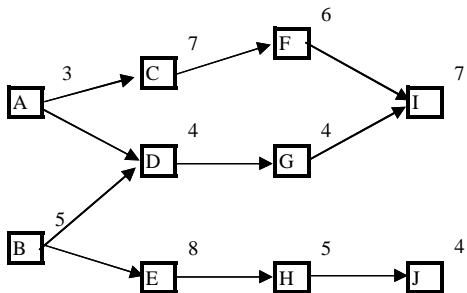
Step 38

Congratulations. You finished one question.

CPM PROBLEM 1 -- WORKED TWICE

CPM PROBLEM 1-B

Calculate the critical path for the network shown in the following diagram.



Syntax

D

D is the duration

N

N is the node identifier

Solution to Problem 1-B

Step 1

Choose one notational system and stay with it.

Here's an alternate syntax. Not my favorite, but it might be something you like.

WBSID		
Early Start	Duration	Early Finish
Late Start	Float	Late Finish

Step 2

Decide upon an expression format and stay with it.

For this example, I will use Day 1 as the starting day, rather than Day 0.

Also, for this example, I will follow the pattern used in MS Project.

If Node A finishes on Day 4, then Node C will start on the next day ($4+1 = 5$).

The math is a bit more complex, but this notation matches "actual practice" better.

Step 3

Set a starting task. In this example, there are two "starting points".

I prefer to then add a pseudo-node so that there is a single starting point.

The starting "day" for this task is day 1, and the duration for a pseudo-node is 0.

Start		
1	0	

Step 4

The end date for node "Start" is the "early start" + "duration".

Start		
1	0	1

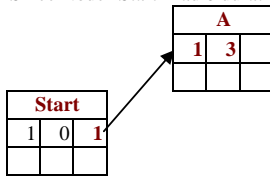
CPM PROBLEM 1 -- WORKED TWICE

Step 5

Add the "next" task, and record the duration.

Then set the "early start" based upon the "early finish" of its "latest" predecessor.

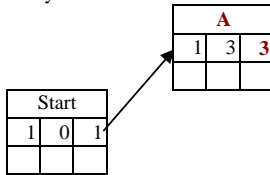
Since Node "Start" had 0 duration, Node "A" can start on the same day as Node "Start".



Step 6

The "early finish" of this task is its "early start" + "duration" - 1.

3 days duration means that work occurs on Day 1, Day 2 and Day 3.

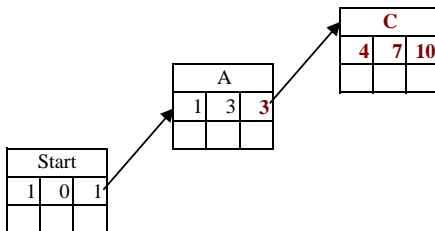


Step 7

Work towards the right, but only add nodes where ALL of the predecessors have already been defined.

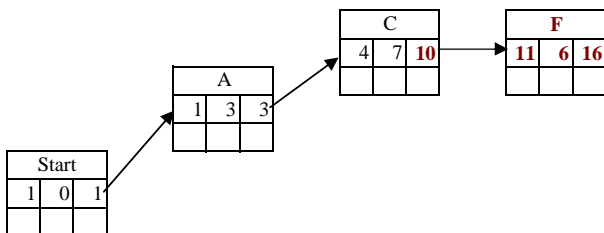
Here the "early start" = the 1 + "early finish" of its "latest" predecessor.

And set the "early finish" of this task is its "early start" + "duration" - 1.



Step 8

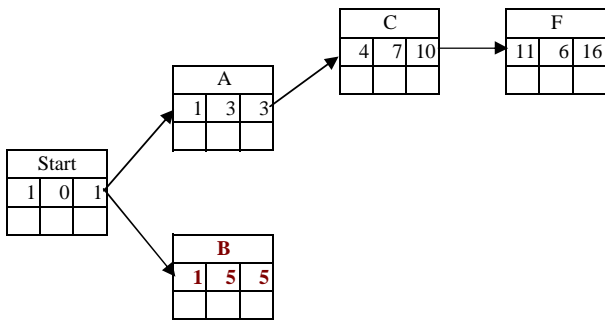
Evaluate Node F.



CPM PROBLEM 1 -- WORKED TWICE

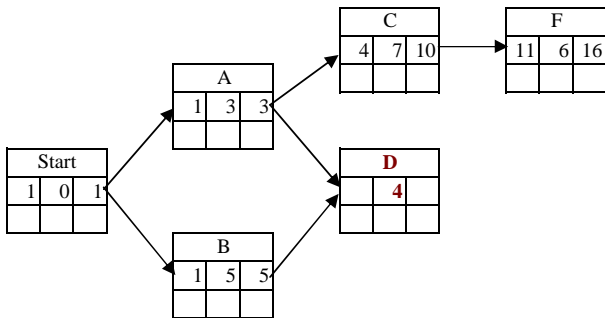
Step 9

Node I has a predecessor (G) that has not yet been defined, so it cannot yet be added.
 Node D has a predecessor (B) that has not yet been defined, so it cannot yet be added.
 Thus, node B needs to be evaluated next.



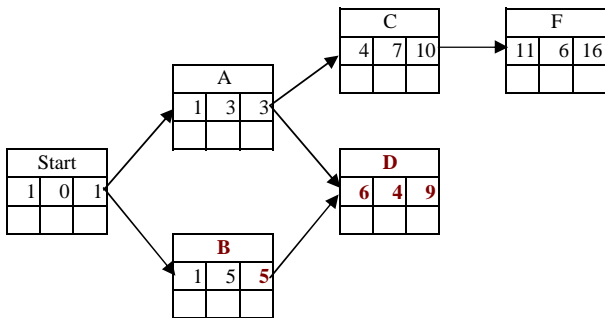
Step 10

Now, node D can be added.



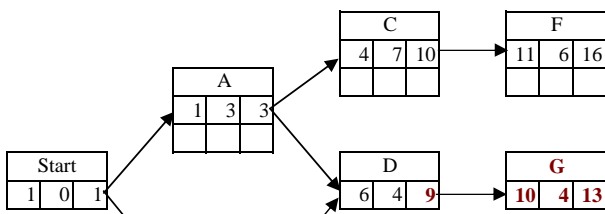
Step 11

Node D has two predecessors.
 Since the relationships show are Finish-to-Start, Node D cannot start until both predecessors have finished.
 Since Node B finishes later than Node A, Node B determines the "early start" for Node D.

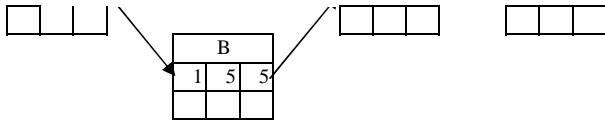


Step 12

Evaluate Node G.

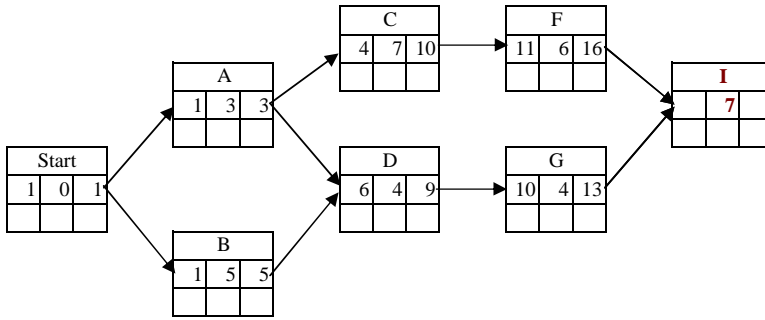


CPM PROBLEM 1 -- WORKED TWICE



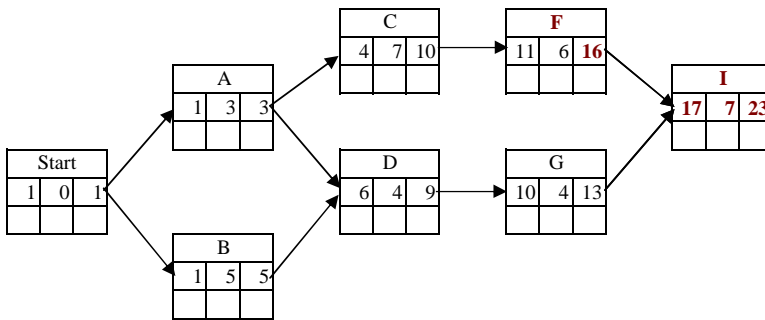
Step 13

Add Node I.



Step 14

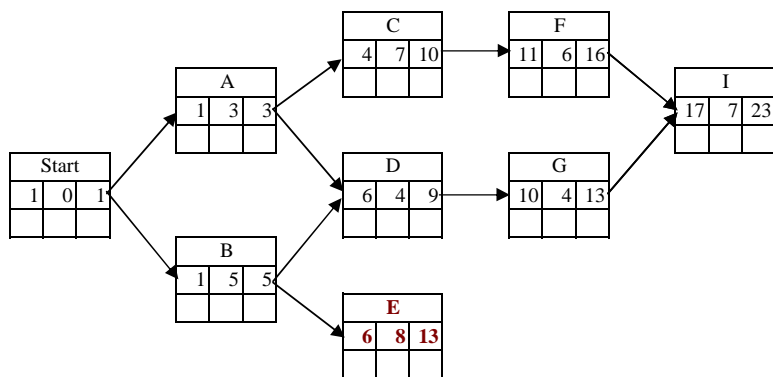
For Node I, the "early start" is the latest "early finish" of its Finish-to-Start predecessors.



CPM PROBLEM 1 -- WORKED TWICE

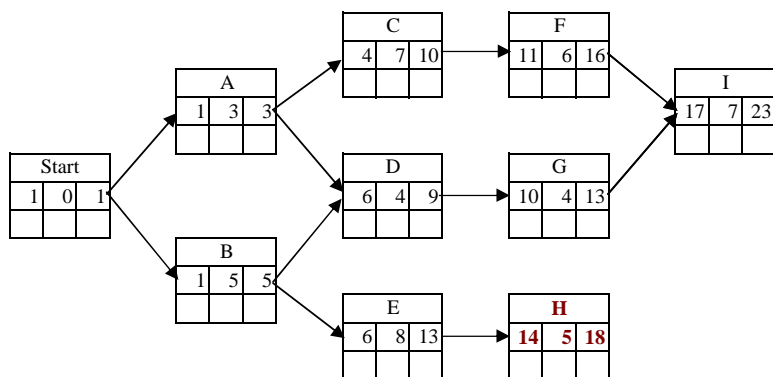
Step 15

Add Node E.



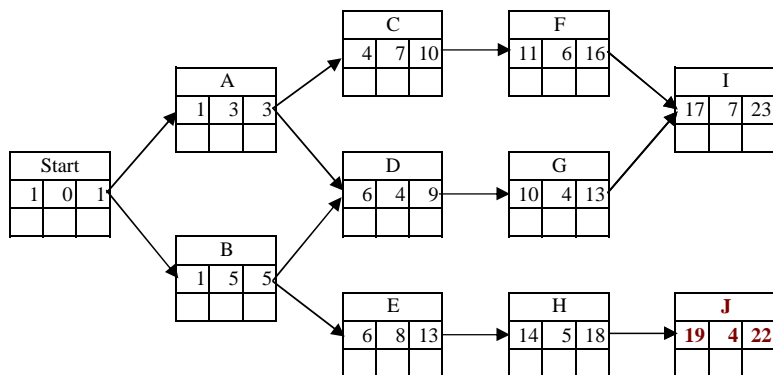
Step 16

Add Node H.



Step 17

Add Node J.

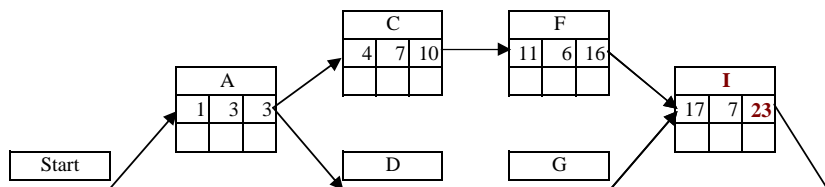


Step 18

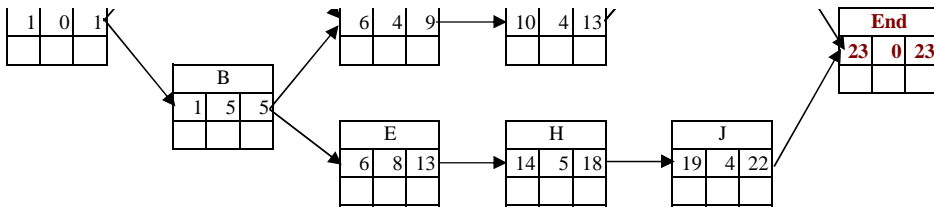
The "backward pass", like the "forward pass" needs a single start point.

So add a pseudo-node for the common "end point".

Then evaluate the "end" node using a duration of 0.

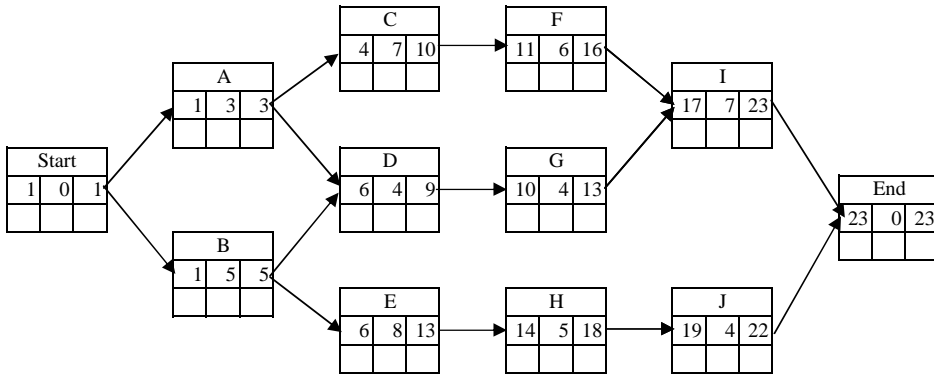


CPM PROBLEM 1 -- WORKED TWICE



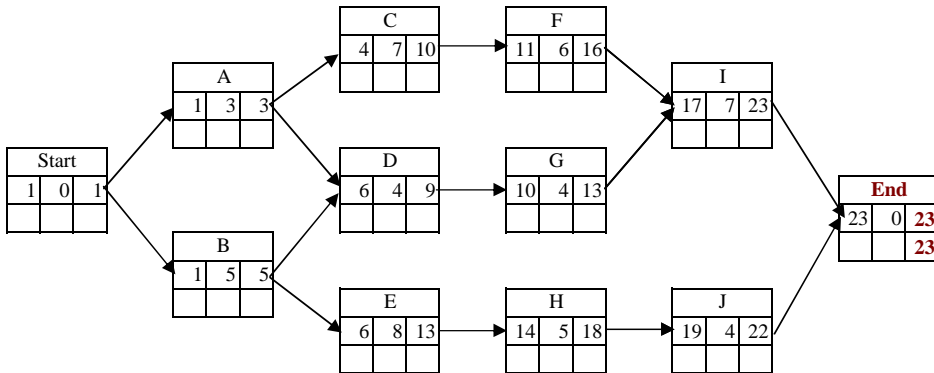
Step 19

STOP HERE: Go back and verify that all of the Node identifications, dependencies and durations have been copied correctly.



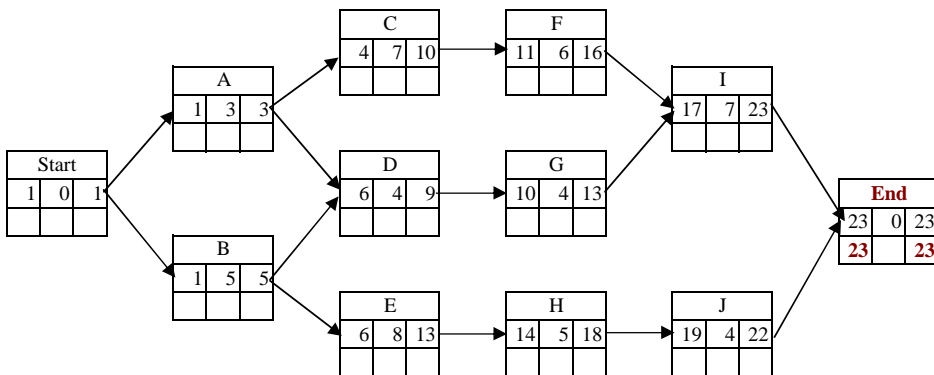
Step 20

The "late finish" for the final task in the network is equal to the "early finish".



Step 21

When the "duration" is 0, the "late start" for a node is the same as the "late finish".

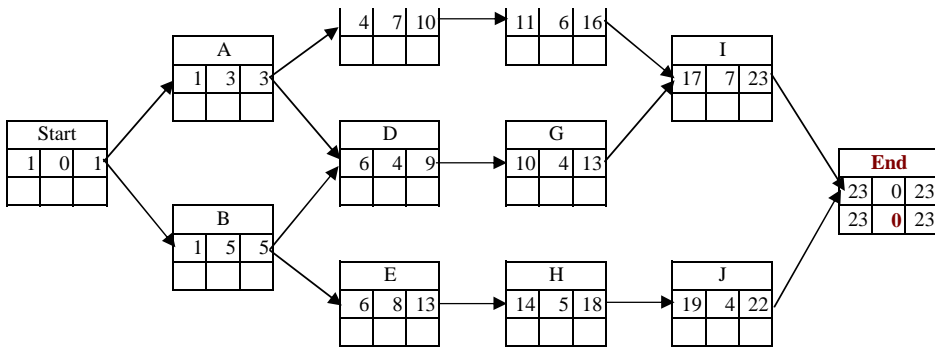


Step 22

The "float" for a node is the "late finish" - "early finish".



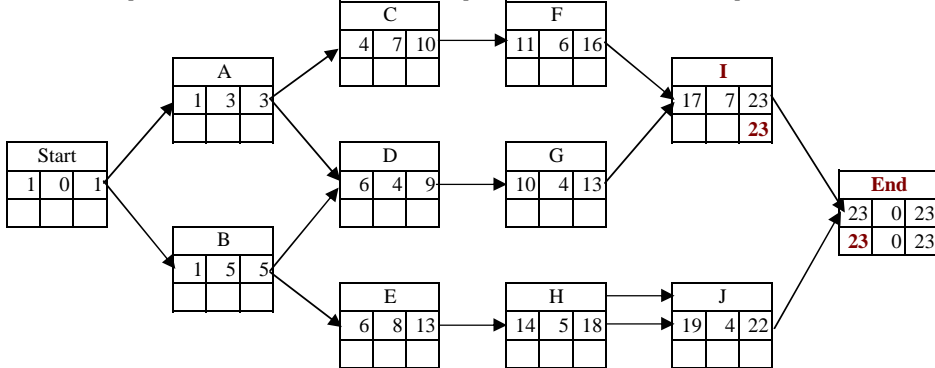
CPM PROBLEM 1 -- WORKED TWICE



Step 23

Since Node "End" has a zero duration, Node "I" can end on the same day that Node "End" begins.

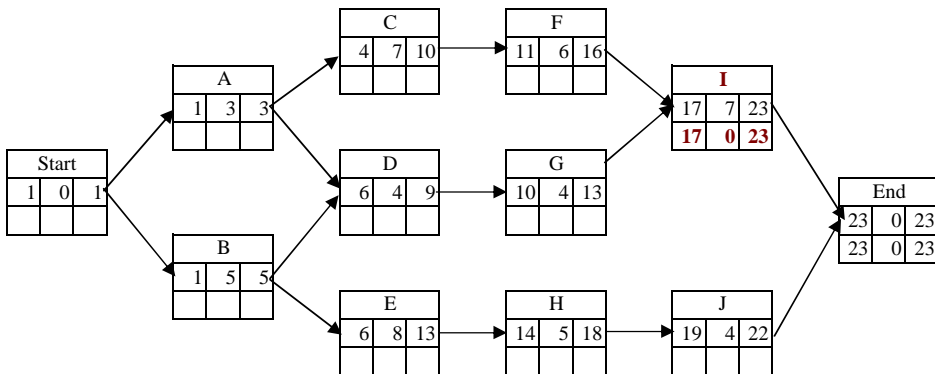
Thus, for the special case of nodes with 0 duration, the predecessor "late finish" = the dependents "late start".



Step 24

The "late start" = "late finish" - "duration" + 1.

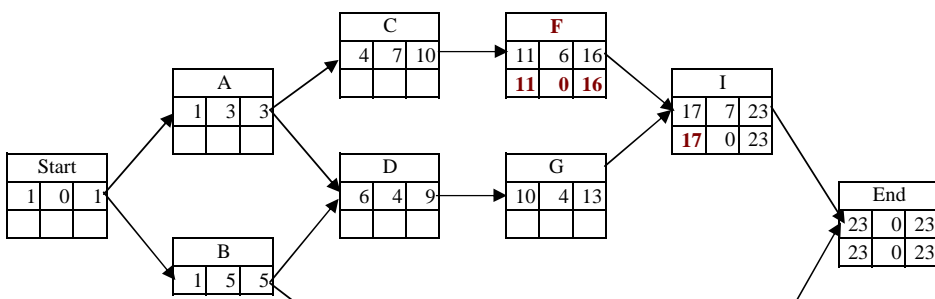
Remember, "float" = "late finish" - "early finish".



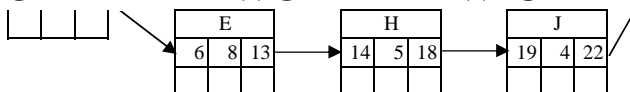
Step 25

Evaluate Node F.

The "late finish" for a node is the earliest ("late start" - 1) from all of the nodes that are dependent on this node.

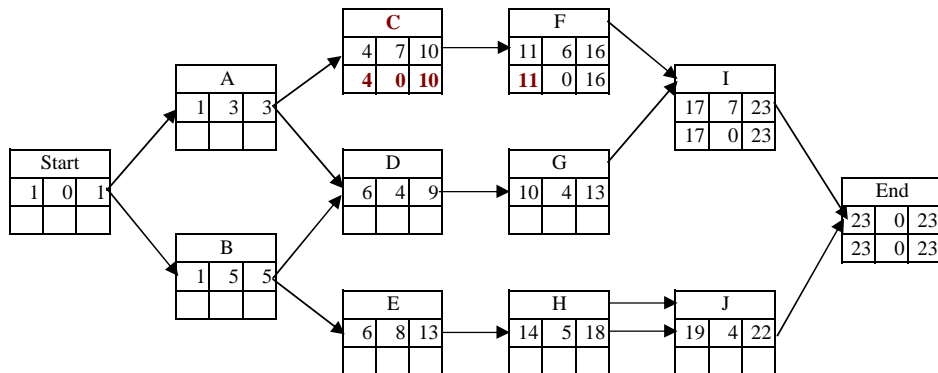


CPM PROBLEM 1 -- WORKED TWICE



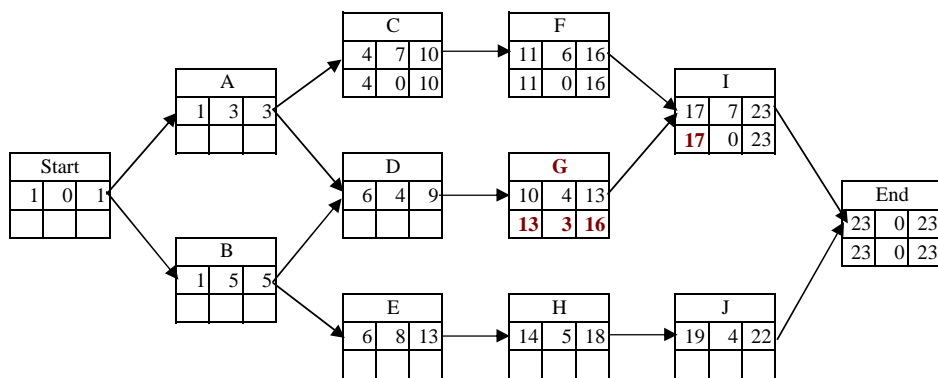
Step 26

Evaluate Node C.



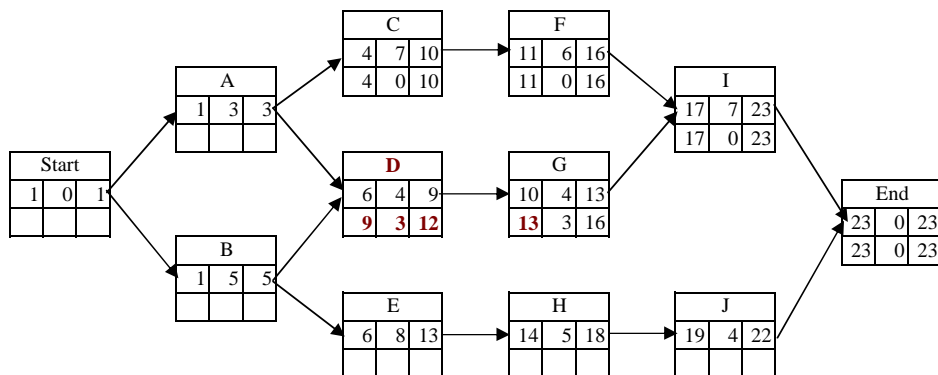
Step 27

Node A cannot be evaluated until both Node C and Node D have been evaluated. So go back and evaluate Node G.



Step 28

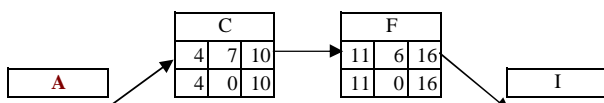
Evaluate Node D.



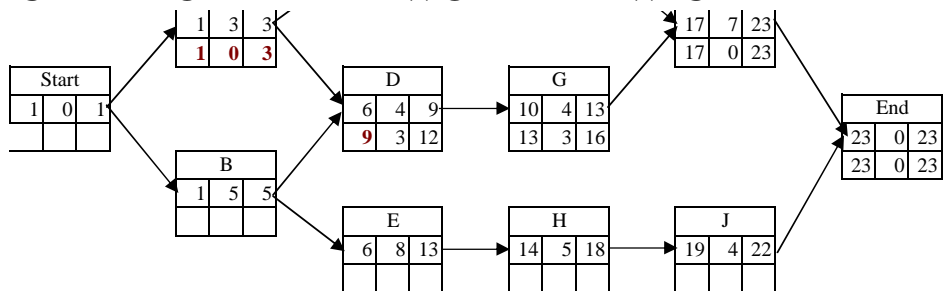
Step 29

Evaluate Node A.

Remember that the "late finish" for a node is the EARLIEST ("late start" - 1) for it's dependents.

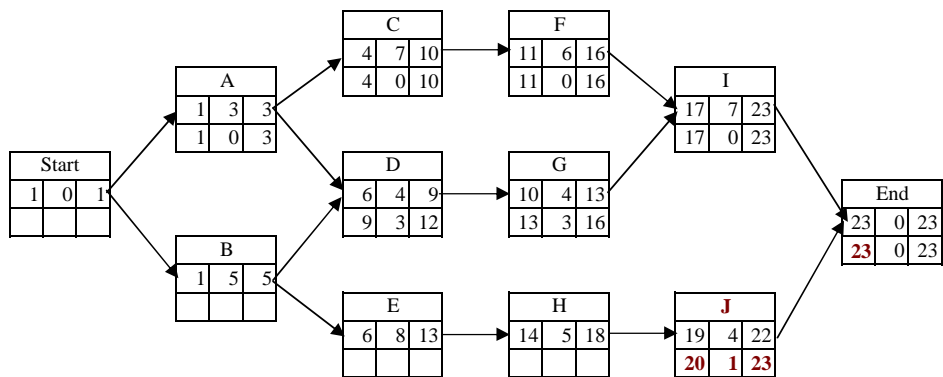


CPM PROBLEM 1 -- WORKED TWICE



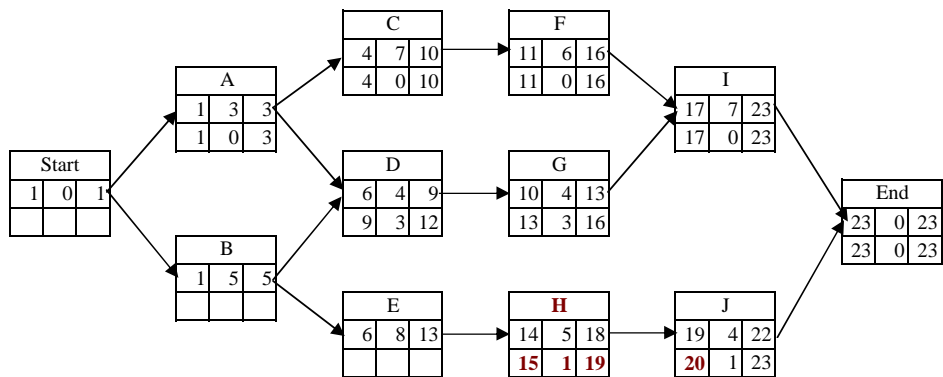
Step 30

Node B cannot be evaluated until the chain J-H-E has been evaluated, so evaluate Node J next.



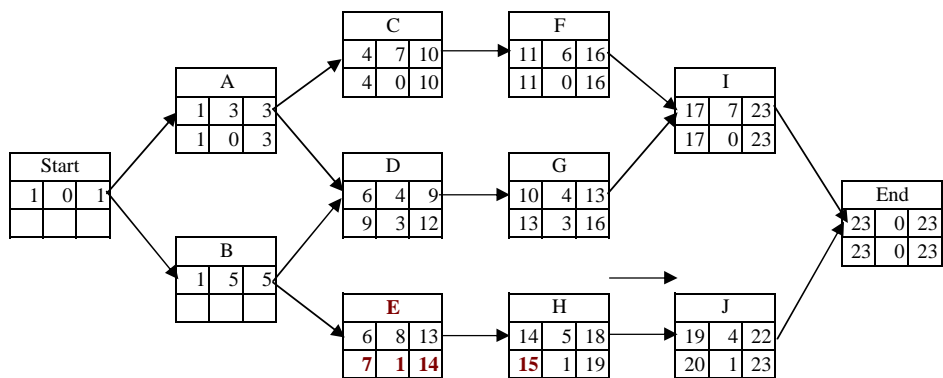
Step 31

Evaluate Node H.



Step 32

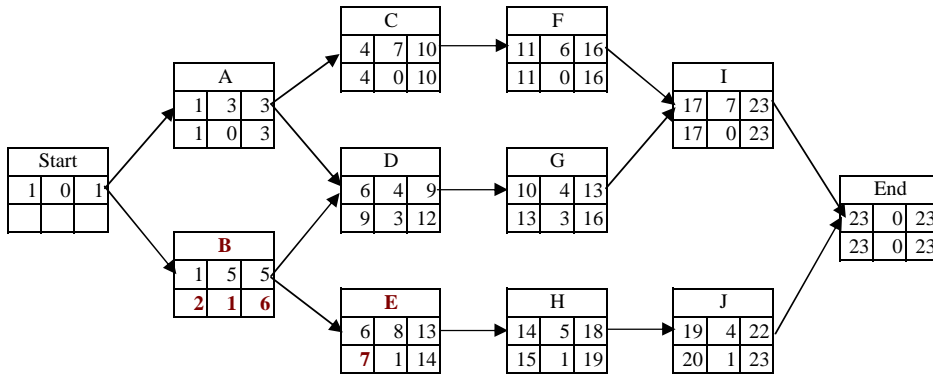
Evaluate Node E.



CPM PROBLEM 1 -- WORKED TWICE

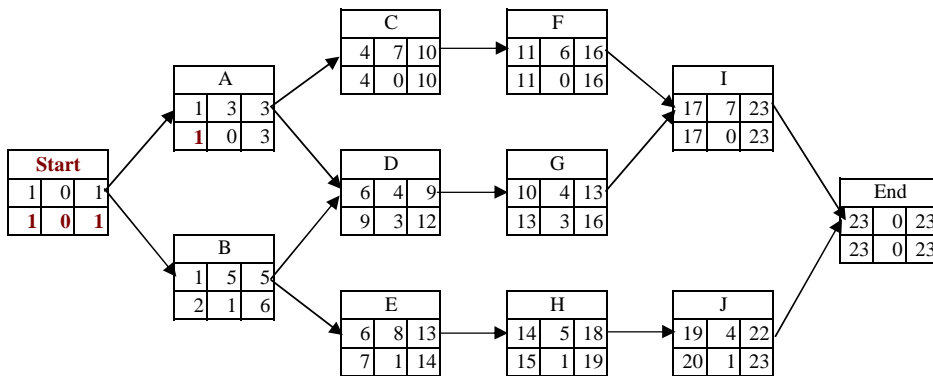
Step 33

Evaluate Node B, using the earliest ("late start" - 1) from it's dependents.



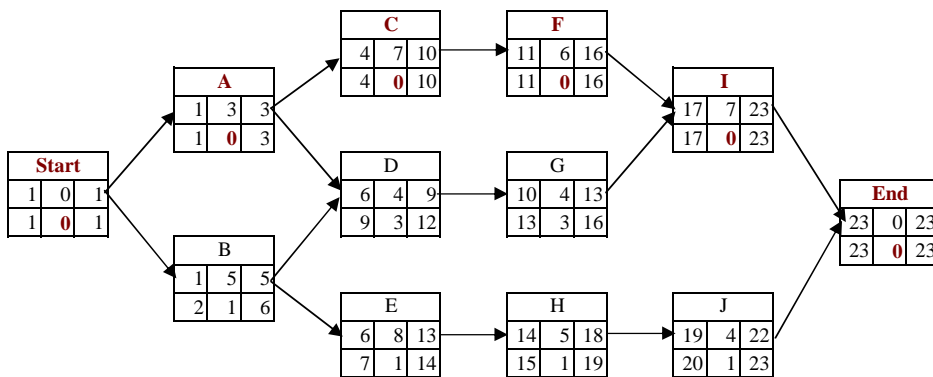
Step 34

Finish the "backward pass" by evaluating the "start" node.



Step 35

Find the path where all of the "float" values are 0.



Step 36

Write down the answer, ignoring the pseudo-nodes "start" and "end".

A → C → F → I

Step 37

Check the possible answers and find a match.

Double check the match. Does it have exactly the same number of nodes, the same beginning, the same ending and all the same values.

Step 38

CPM PROBLEM 1 -- WORKED TWICE

Compare the results using Method A and Method B.

Both calculate the same critical path.

Both calculate the same values for the float.

Both calculate the same "early finish" and "late finish" dates.

Method A is easier to do mathematically because there are no special cases.

Method B, however, more closely mimics project reality.

If you are studying CPM to prepare for a post-apoplectic world without electronic planning tools, then Method B is better.

If, however, you are studying CPM to pass the PMP test, then Method A is easier, and it results in the same answer.

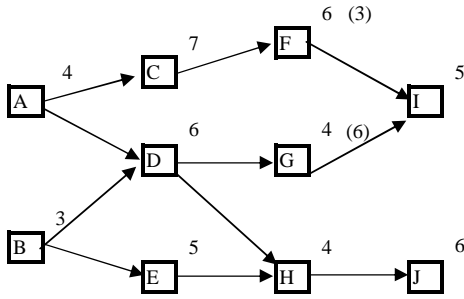
Copyright Robert E. Perrine, PMP

<mailto:rperrine5@cox.net>

<http://members.cox.net/rperrine5>

CPM PROBLEM 2 -- WITH A TWIST

Use the network diagram, shown below, to determine the net effect on project duration if a resource is transferred. Due to this transfer the duration of Node G will increase from 4 periods to 6 periods, while the duration of Node F will decrease from 6 periods to 3 periods.



Syntax
 D is the duration
 N is the node identifier

Solution to Problem 2

Step 1

Choose one notational system and stay with it.

WBSID	
Duration	Float
Early Start	Early Finish
Late Start	Late Finish

Step 2

Decide upon an expression format and stay with it.

I prefer to start with day=0.

I also find it simpler to let the finish date for one task be the start date for the dependent tasks.

Step 3

Set a starting task. In this example, there are two "starting points".

I prefer to then add a pseudo-node so that there is a single starting point.

The starting "day" for this task is day 0, and the duration for a pseudo-node is also 0.

Start	
0	
0	

Step 4

The end date for a node is the "early start" + "duration" for that node.

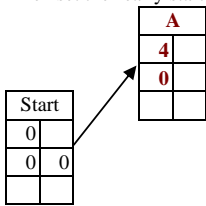
Start	
0	
0	0

CPM PROBLEM 2 -- WITH A TWIST

Step 5

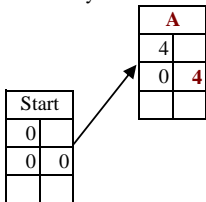
Add the "next" task, and record the duration.

Then set the "early start" = the "early finish" of it's "latest" predecessor.



Step 6

The "early finish" of this task is it's "early start" + "duration".

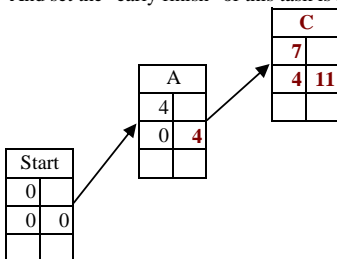


Step 7

Work towards the right, but only add nodes where ALL of the predecessors have already been defined.

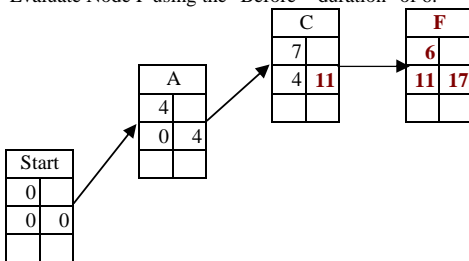
Remember: set the "early start" = the "early finish" of it's "latest" predecessor.

And set the "early finish" of this task is it's "early start" + "duration".



Step 8

Evaluate Node F using the "Before" "duration" of 6.



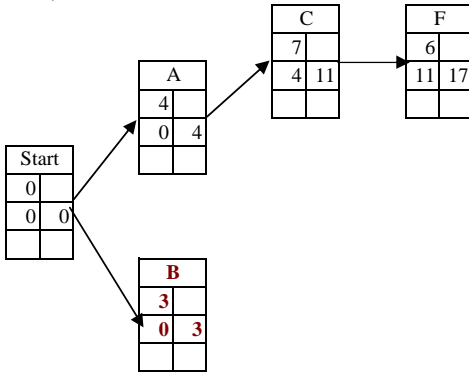
CPM PROBLEM 2 -- WITH A TWIST

Step 9

Node I has a predecessor (G) that has not yet been defined, so it cannot yet be added.

Node H has a predecessor (B) that has not yet been defined, so it cannot yet be added.

Thus, node B needs to be evaluated next.

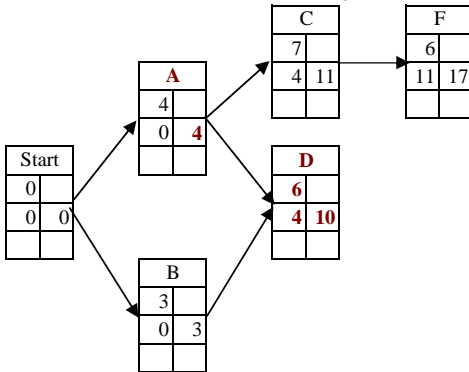


Step 10

Node D has two predecessors.

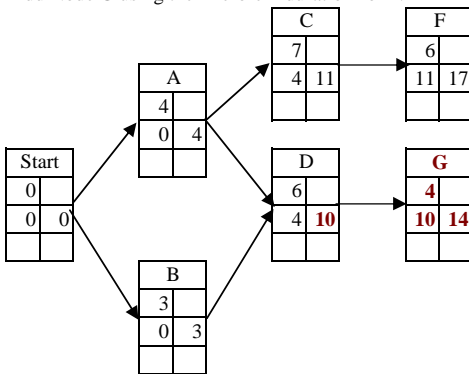
Since the relationships show are Finish-to-Start, Node D cannot start until both predecessors have finished.

Since Node A finishes later than Node B, Node A determines the "early start" for Node D.



Step 11

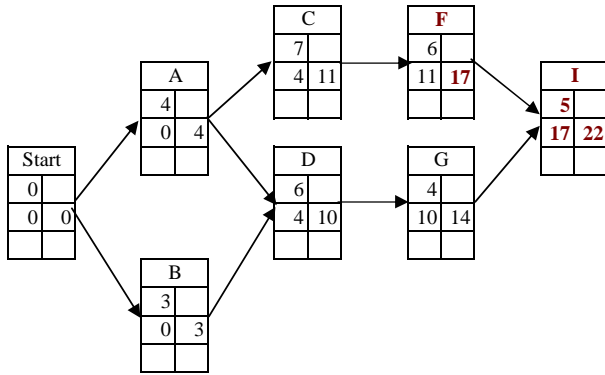
Add Node G using the "Before" "duration" of 4.



CPM PROBLEM 2 -- WITH A TWIST

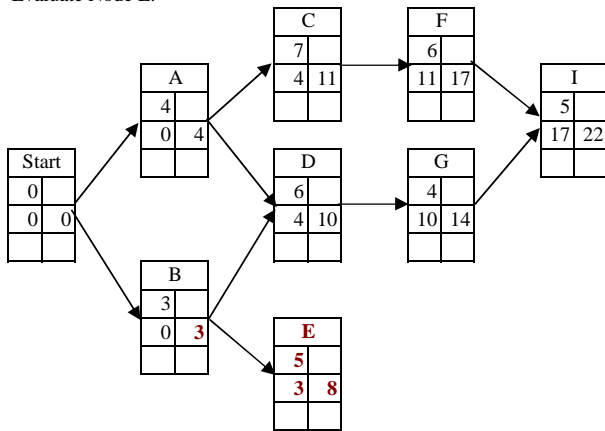
Step 12

Evaluate Node I.



Step 13

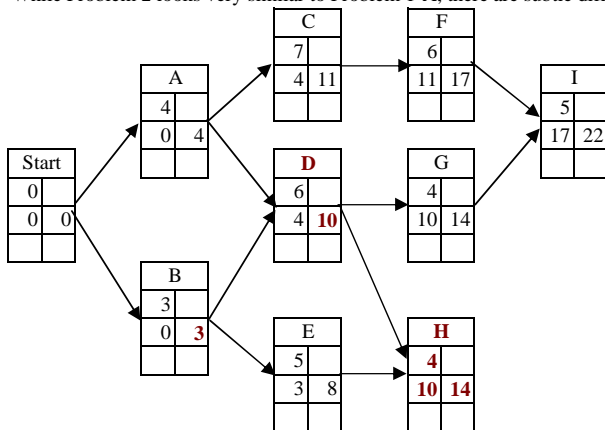
Evaluate Node E.



Step 14

Evaluate Node H. Be aware, however, that Node H has two predecessors.

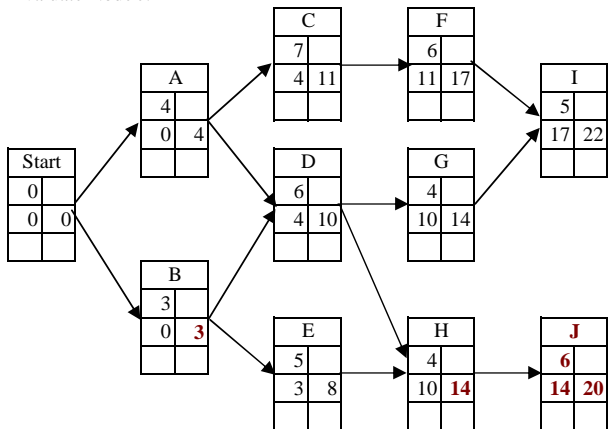
While Problem 2 looks very similar to Problem 1-A, there are subtle differences.



CPM PROBLEM 2 -- WITH A TWIST

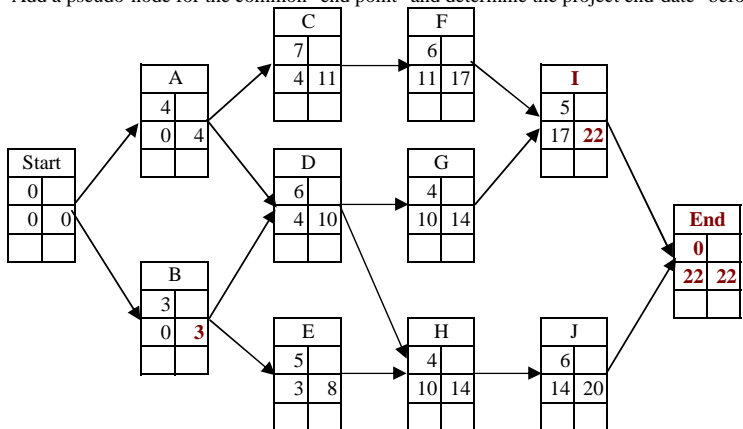
Step 15

Evaluate Node J.



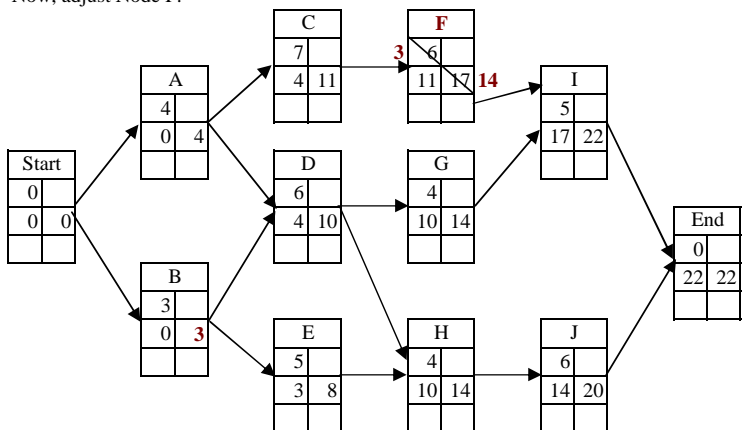
Step 16

Add a pseudo-node for the common "end point" and determine the project end-date "before" the change occurs.



Step 17

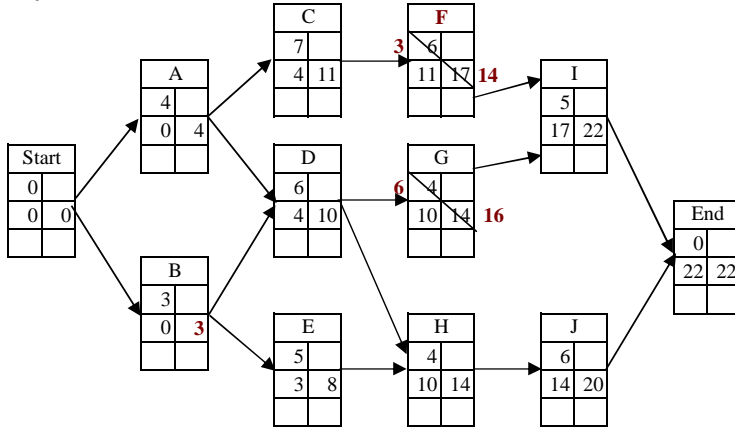
Now, adjust Node F.



CPM PROBLEM 2 -- WITH A TWIST

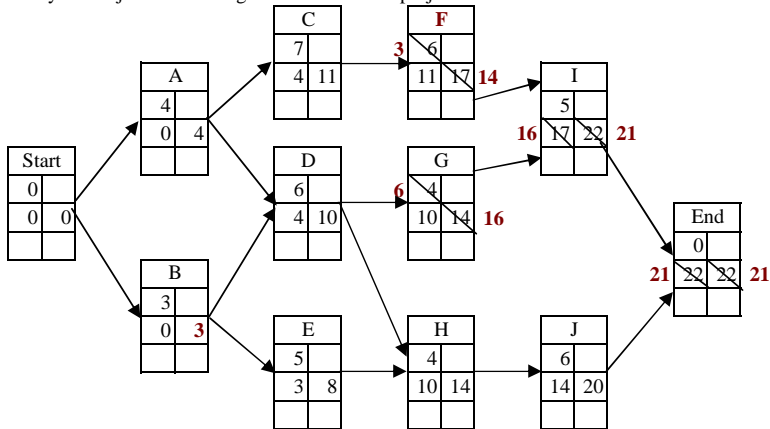
Step 18

Adjust Node G.



Step 19

Carry this adjustment through to the end of the project.



Step 20

Evaluate the results.

- 22 Prior end date
- 21 New end date
- 1 Project end date is brought in by 1 "period" (day/week/month/year).

Step 21

Check the possible answers and find a match.

Double check the match. Does the wording imply that -1 means a reduction, or is +1 a reduction?

Step 22

Congratulations. You finished another question.

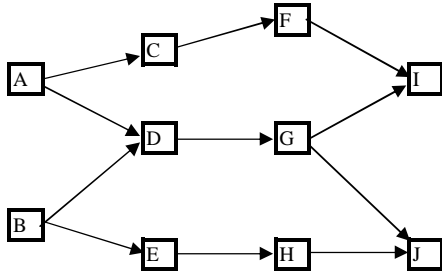
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CPM PROBLEM 3 -- PMP STYLE

You are the project manager for a complex software development project. This project is using a new language with which very few programmers are proficient. An expert has been located to assist in training. He has already begun work and is very familiar with the team and the project requirements. Due to his expertise he is in constant demand. You were very lucky to get him in to assist in the training. Suddenly, however, his next project is cancelled. He approaches you and offers to assist with the work on Node F, G or H. His commitments elsewhere mean that he can only work on one of these tasks. Your company wants this project completed early, and the early completion incentive will more than compensate for the payment to this expert. Use the network diagram and the evaluation criteria show below. Where should you assign this resource, and what will this do to the project duration?



Task	Duration Estimate	Revised Estimate
A	3	
B	5	
C	4	
D	5	
E	6	
F	7	4
G	5	3
H	8	5
I	8	
J	3	

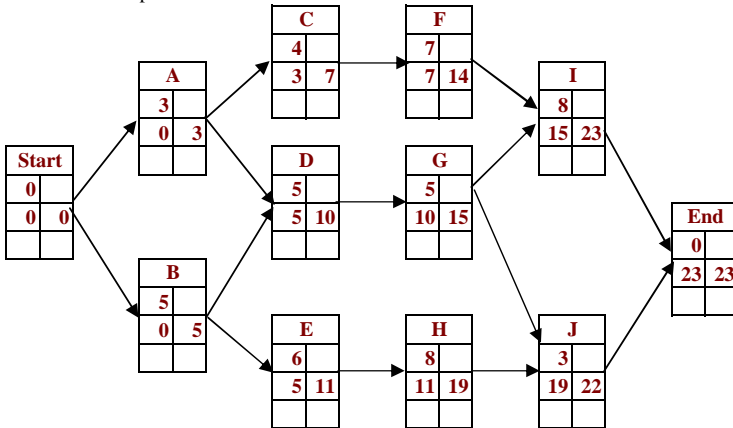
Syntax

N N is the node identifier

Solution to Problem 3

Step 1

Do the forward pass.



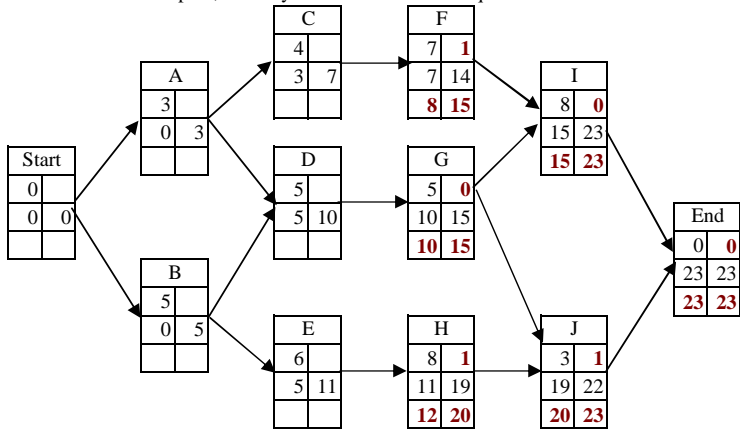
Step 2

STOP HERE: Go back and verify that all of the Node identifications, dependencies and durations have been copied correctly.

CPM PROBLEM 3 -- PMP STYLE

Step 3

Work the backward pass, but only as far as the nodes in question.



Step 4

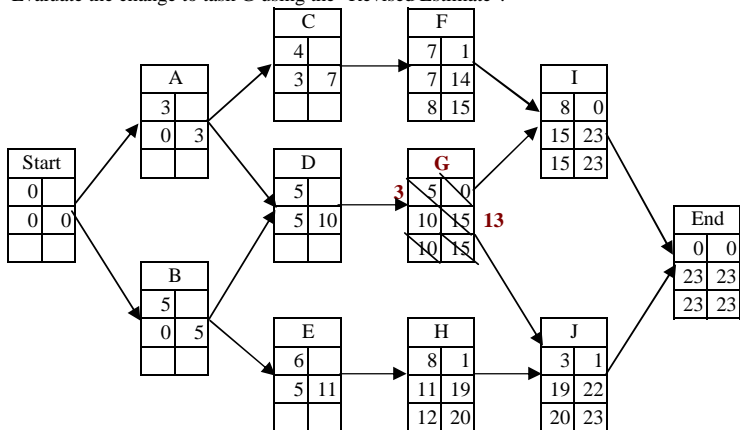
Adding a resource will only reduce project duration if the resource is added to a critical path task.

Note that tasks F and H are not on the critical path, but task G is.

Thus, the appropriate use of the resource is to add this expert to task G.

Step 5

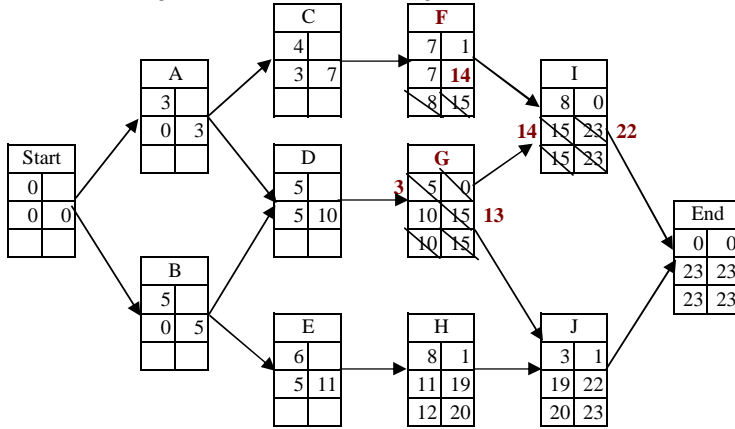
Evaluate the change to task G using the "Revised Estimate".



CPM PROBLEM 3 -- PMP STYLE

Step 6

Evaluate the change on task I. Note that even though task G now ends at "13", task F will still not finish until "14".

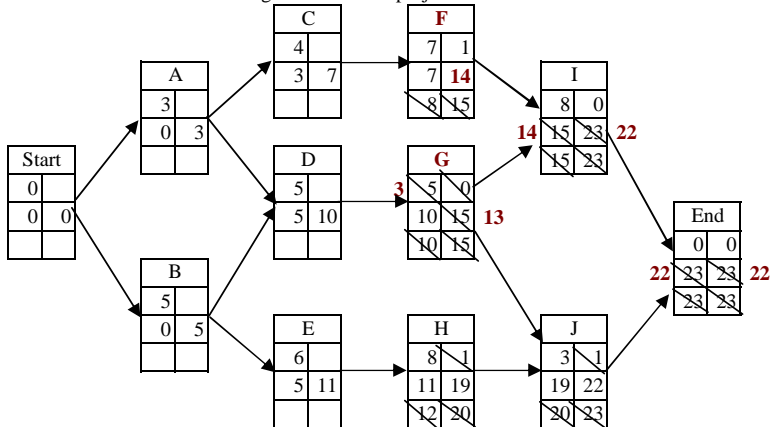


Step 7

Note that the change in task G will have no effect on task J because task H already finished later than task G.

Step 8

Evaluate the effect of the change in G-I on the project end date.



Step 9

Evaluate the results.

- 23 Prior end date
- 22 New end date
- 1 Project end date is brought in by 1 "period" (day/week/month/year).

ANSWER: Adding the expert to Task G will reduce the project duration by 1 "period".

Step 10

Check the possible answers and find a match.

Double check the match. Does the wording imply that -1 means a reduction, or is +1 a reduction?

Step 11

Congratulations. You finished another question.

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<http://members.cox.net/rperrine5>

IMAGES FOR THE CPM PRESENTATION

Styles

WBSID	
Duration	Float
Early Start	Early Finish
Late Start	Late Finish

WBSID		
Early Start	Duration	Early Finish
Late Start	Float	Late Finish

Starting Point

Task A	
Duration = 4	
Early Start = 0	

Task A	
Duration = 4	
Early Start = 1	

Addition

$$\text{Early Finish} = \text{Early Start} + \text{Duration}$$

Task A	
Duration = 4	
Early Start = 0	Early Finish=4

$$\text{Early Finish} = \text{Early Start} + \text{Duration} - 1$$

Task A	
Duration = 4	
Early Start = 1	Early Finish=4

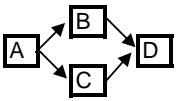
Increment

Task A	
4	
0	4

Task B	
7	
ES ?	

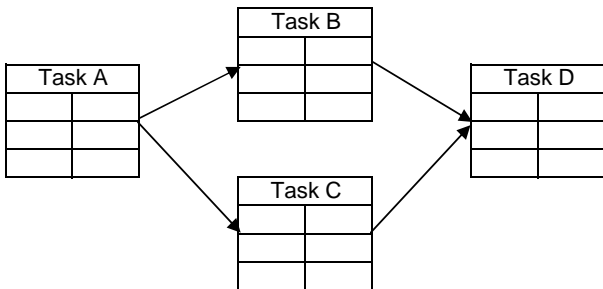
Early Start = Early Finish of predecessor
OR
Early Start = 1 + Early Finish of Predecessor

Problem 1

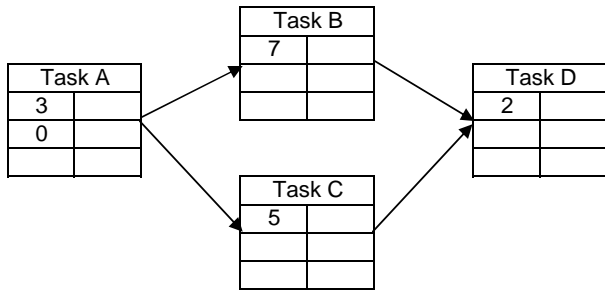


Durations	
A	3
B	7
C	5
D	2

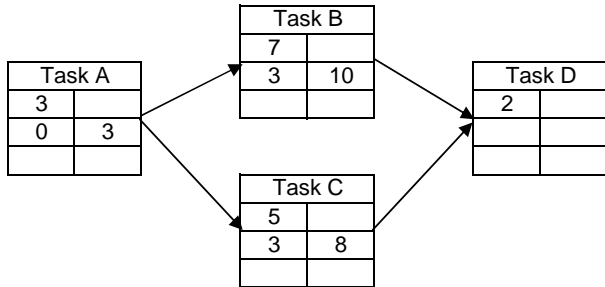
Q1 Drawing



Q1 Known Values

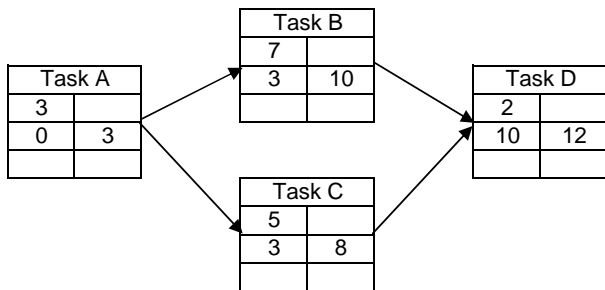


Q1 Flow



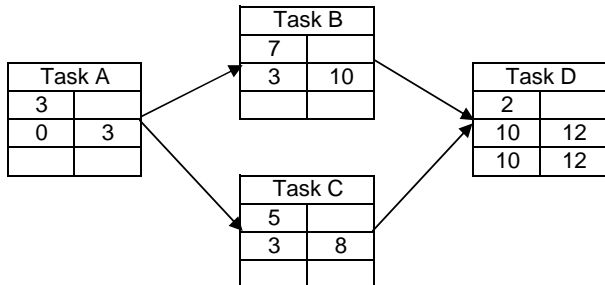
Task	Variable	Equation
A	Early Finish	Early Finish = Early Start + Duration $3 + 0 \rightarrow 3$
B	Early Start	Early Start = Early Finish of predecessor 3
B	Early Finish	Early Finish = Early Start + Duration $3 + 7 \rightarrow 10$
C	Early Start	Early Start = Early Finish of predecessor 3
C	Early Finish	Early Finish = Early Start + Duration $3 + 5 \rightarrow 8$

Q1 Join



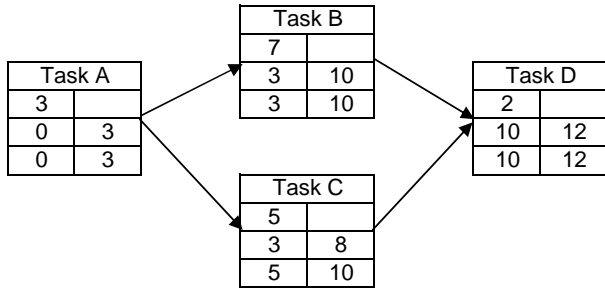
Task	Variable	Equation
D	Early Start	Early Start = Early Finish of LATEST predecessor 10
D	Early Finish	Early Finish = Early Start + Duration $10 + 2 \rightarrow 12$

Q1 End Node



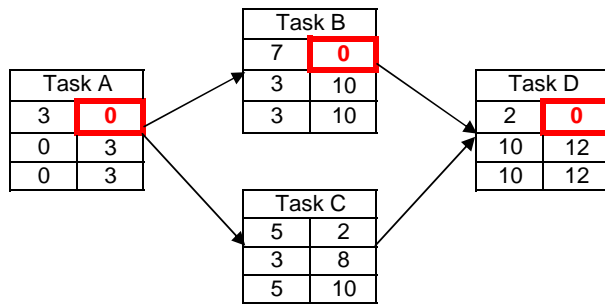
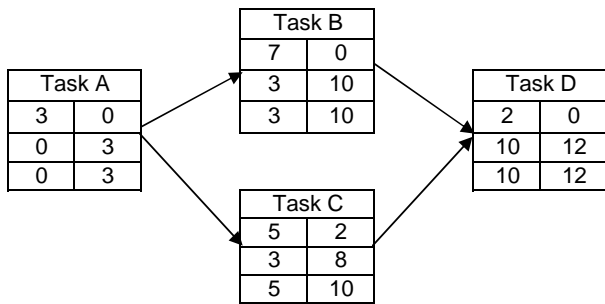
Task	Variable	Equation
D	Late Finish	Late Finish = Early Finish for the FINAL node 12
D	Late Start	Late Start = Late Finish - Duration $12 - 2 \rightarrow 10$

Q1 Backwards



Task	Variable	Equation
B	Late Finish	Late Finish = Late Start of EARLIEST dependent 10
B	Late Start	Late Start = Late Finish - Duration $10 - 7 \rightarrow 3$
C	Late Finish	Late Finish = Late Start of EARLIEST dependent 10
C	Late Start	Late Start = Late Finish - Duration $10 - 5 \rightarrow 5$
A	Late Finish	Late Finish = Late Start of EARLIEST dependent 3
A	Late Start	Late Start = Late Finish - Duration $3 - 3 \rightarrow 0$

Q1 Forward

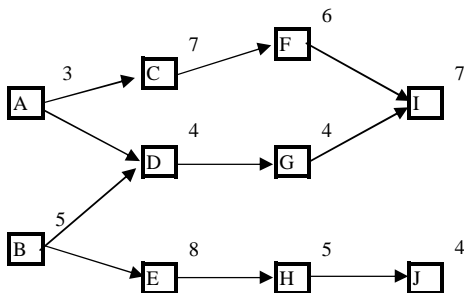


Wrap-Up

WBSID	
Duration	Float
Early Start	Early Finish
Late Start	Late Finish

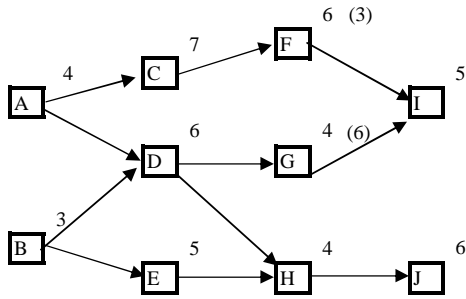
Moderately Complex Question

Find the critical path for this network diagram.



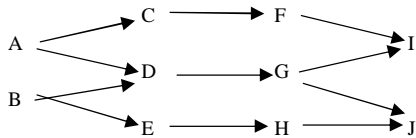
Take Home Question #2

Use the network diagram, shown below, to determine the net effect on project duration if a resource is transferred. Due to this transfer the duration of Node G will increase from 4 periods to 6 periods, while the duration of Node F will decrease from 6 periods to 3 periods.



Take Home Question #3

You are the project manager for a complex software development project. This project is using a new language with which very few programmers are proficient. An expert has been located to assist in training. He has already begun work and is very familiar with the team and the project requirements. Due to his expertise he is in constant demand. You were very lucky to get him in to assist in the training. Suddenly, however, his next project is cancelled. He approaches you and offers to assist with the work on Node F, G or H. His commitments elsewhere mean that he can only work on one of these tasks. Your company wants this project completed early, and the early completion incentive will more than compensate for the payment to this expert. Use the network diagram and the evaluation criteria show below. Where should you assign this resource, and what will this do to the project duration?



Task	Duration Estimate	Revised Estimate
A	3	
B	5	
C	4	
D	5	
E	6	
F	7	4
G	5	3
H	8	5
I	8	
J	3	

Problem 2 Show with Start and End nodes

